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Optimized joint bandwidth and playout control for streaming-traffic over wireless-channels

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Abstract

Media-streaming applications over time-varying faded wireless channels present multiple still open challenges, mainly arising from the energy-limited nature of the wireless connections, as well as the delay and delay-jitter sensitive features of the conveyed media traffic. In this paper, we recast the challenges to be tackled in the form of a suitable non-linear constrained optimization problem involving finite-capacity G/G/1 queues at both transmit and receive nodes of the considered connection. Afterwards, we develop an optimized scheduling policy that seeks to improve media-streaming performance by building up adaptive joint control of the media source rate (e.g., the connection bandwidth), channel rate (e.g., transmit rate) and delivery rate (e.g., playout rate). Formally speaking, the objective that we consider is the maximization of the average transmit rate, when constraints on the: *i)* allowed average and peak energies; *ii)* maximum connection bandwidth (e.g., maximum source rate); and: *iii)* buffer' capacities are simultaneously present. Furthermore, to properly cope with the jitter-sensitive feature of media-streaming, we also require that the joint controller to be developed is able to guarantee jitter values below any desired target limit. The joint controller we derive as solution of the described optimization problem works in an *adaptive* (e.g., time-varying) and *Cross-Layer* (CL) way, and requires synergic cooperation of the Application (APP), Data Link (DL) and Physical (PHY) layers of the underlying protocol stack. The presented numerical results give insight about the attained optimized tradeoff among transmit rate, delay and delay-jitter.

Keywords: `Multimedia wireless connections, cross-layer optimization, adaptive playout, queuing systems.`

AMS Subject Classification: `05A16, 65N38, 78M50`

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1. Introduction and goals.

Over last years, a strong proliferation in the use of multimedia technology has been experienced, triggered by several reasons. First, wireless networking architectures (i.e., media Content Delivery Networks (CDNs) and Wireless Local Area Networks (WLANs)) are becoming an integral part of communication environments [1, Chaps.5,6,7]. Second, the utilization of new hand-held wireless devices (such as, media phones, PDAs and laptops) is becoming common. Third, new multimedia applications (i.e., streaming stored/live audio/video) become popular, first in the Internet wired environment and now in wireless mobile environment [2, Chaps.14,15].

However, these wireless media opportunities bring with them several technological challenges. In fact, wireless networks suffer from large variations in connection conditions, mainly due to fading phenomena, users' mobility, co-channel and multiple-access interferences. Since currently deployed network protocols (as, for example, the Internet one [1, Chap.4]) offer best-effort services to the higher layers of the protocol stack, the Application and/or Transport layers need to implement adaptive bandwidth control mechanisms, in order to sustain reliable connections able to support delay and delay-jitter sensitive media applications [2]. The focus of this paper is on the maximization of the media-streaming performance in terms of average transmit rate under constraints on the energy-budget available at the PHY layer, transmit buffer capacity and playout buffer capacity at the DL layer and maximum bandwidth allowed at the APP/Transport layers of the considered protocol stack.

1.1. *Tackled problem and related works.*

Specifically, according to Fig.1, we model the transmit node as a time-slotted fluid G/G/1 queuing system fed by a source (e.g., a Variable Bit Rate (VBR) Media Encoder) whose output rate (e.g., the connection bandwidth) may be controlled. The receive node is modelled as a G/G/1 queue able to implement adaptive control of the playout rate. In this operating scenario, the problem we go to tackle deals with the *closed-form* (e.g, no iterative) *cross-layer* (e.g., queues *and* channel-state aware) design of the controller that *jointly* performs optimal adaptive management of the connection bandwidth, transmit rate and radiated energy, when five system's constraints are simultaneously present. The first constraint is on the available energy per slot *averaged* over the fading statistics of the underlying wireless channel of Fig.1 and *conditioned* on the current queue-states. This constraint arises from energy limitations typically imposed by the PHY layer of the considered communication system of Fig.1 [3]. The second con-

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straint we consider is on the allowed peak-energy per slot. It is still dictated by the PHY layer and arises from spectral compatibility issues and/or limitations on the maximum rate allowed by the considered wireless standard [1, Chap.6]. A third constraint arises from the APP layer and limits the maximum instantaneous bandwidth allowed the connection. The fourth and fifty constraints involve the DL layer and (upper) limit the available capacities (i.e., sizes) of the transmit and playout buffers (see Fig.1).

Overall, the resulting constrained optimization problem we go to tackle is an instance of *optimal resource allocation* problem for delay-sensitive media-content delivery over energy-limited faded-channels, that directly involves the APP, DL and PHY layers of the underlying system's protocol stack.

In this context, optimized joint energy/queue control policies derived by exploiting the analytical tool of the Markov Decision Process (MDP) and, thus, implemented via Dynamic Programming are presented, for example, in [4], [5], [6], [7], [9]. Specifically, in [4] a *discrete-state* faded link is considered and the structural properties of the scheduling policy that optimizes the energy-vs-delay tradeoff are provided. [7] resorts to the queuing-theory [8] to develop optimized scheduling policies for real-time traffic under hard constraints on packet deadlines. However, these contributions do *not account* for the constraints on the allowed average and/or peak energies and focus on *finite-state* Gilbert-Elliot type fading channels.

By fact, even if *adaptive* playout control is recognized to be a key element for enhancing media-streaming performance, in the current literature it is designed off-line, e.g., without pursuing an adaptive approach. Almost all the works of the current literature do not consider, indeed, the optimization problem from a joint CL point of view and do not take into account for the time varying nature of the underlying wireless channel. Furthermore, they often consider specific queuing models (as the M/G/1 one), or resort to complex to implement iterative solutions based on the Dynamic Programming [2], [18]. Along this line, in [10] authors explore the impact of the playout buffer capacity on the rendered media quality in video streaming applications. They show that a large buffer generally improves video quality, but incurs in large buffering delays, that, in turn, may be no tolerated in delay and delay-jitter sensitive streaming applications. [10] develops a architectural model for video streaming and derives an analytical expression for the minimum playout buffer capacity required to achieve a target video quality. Authors of [11] present an analytical model based on the M/G/1 queueing system, and examine the effects of finite buffer capacity on the resulting delay-jitter. In [12], authors develop a policy for controlling both transmit power and playout scheduling, in order to minimize power consumption and maximize the media playout quality. They also provide structural properties of the obtained solution via a Dynamic Programming approach. However, they consider *pre-stored* media contents, while in our work we jointly control in *real-time* source, transmit and playout rates. In [13], the authors consider a multiuser media system for streaming over randomly time-variant channels. Specifically, in [13] each user is equipped with a playout buffer of *infinite* capacity for storing the received packets. The objective considered in [13] is to allocate (on a per slot basis) transmit power, so to minimize the average transmit power under two constraints. The first constraint is on the peak power available at each slot, while the second one requires that any underflow of the playout buffer is avoided. Nevertheless, [13] differs from our work under several aspects. First of all, the objective function considered in [13] is the minimization of the average transmit power conditioned on the fading channel and queue lengths. Furthermore, neither source rate control, nor transmit rate control are considered and the receiver is not able to adapt its playout rate. Moreover, authors of [13] solve their problem by using a Dynamic Programming based approach, so that the resulting controller is *not* in closed-form.

In the sequel, we develop *closed-form* expressions for the optimal controller solution of the tackled *cross-layer* constrained optimization problem. Afterwards, we numerically test actual performance of the proposed controller in terms of average transmit rate, delay and delay-jitter.

The rest of this paper is organized as follows. After introducing in Sect.2 the considered communication architecture and problem setup, in Sect.3 we develop the optimal controller. In Sect.4, we numerically test actual performance of the proposed controller in terms of average transmit rate, delay and delay-jitter. Finally, in Sect.5 we present some conclusive remarks.

About the adopted notation, underlined letters denote vectors, scalar random variable are denoted by bold characters, while their outcomes are indicated by the corresponding no bold symbols. Furthermore, $\mathrm{E}\{.\}$ is the expectation operator, $\mathbb{R}_0^+$ is the set of the nonnegative real numbers, $\mathbb{R}^+$ is the set of the strictly positive real numbers, $\triangleq$ means "equal by definition", while $[x]^+$ indicates the $\max\{x, 0\}$, and $p_\sigma(\sigma)$ is the probability density function (pdf) of the r.v. $\boldsymbol{\sigma}$. Finally, $\mathrm{E}_\sigma\{\varphi(\boldsymbol{\sigma}; s)\} \triangleq \int \varphi(\sigma; s)p_\sigma(\sigma)d\sigma$ denotes the expectation of the bi-argumental function $\varphi(\sigma; s)$ carried out only over the pdf of the r.v. $\boldsymbol{\sigma}$, while $[f(x)]_a^b$ indicates the $\max\{a; \min\{f(x); b\}\}$.

2. System's architecture and problem setup.

The system's architecture we refer to is depicted in Fig.1. Specifically, we consider a fading wireless channel going from a transmit node (e.g., the

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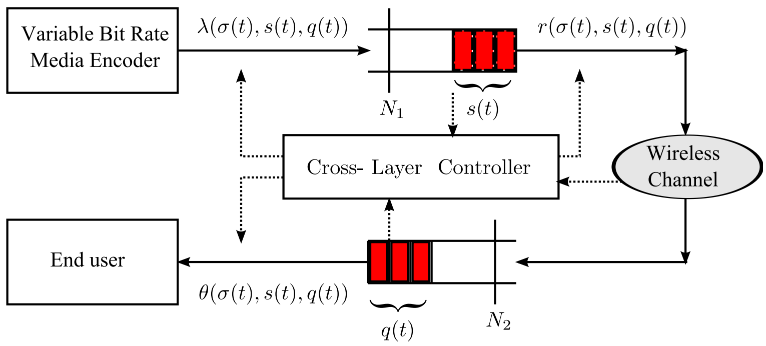

Figure 1. The considered communication architecture for wireless media-streaming.

server node) to a receive node (e.g., the client terminal). Time is slotted, with the t-th slot spanning the (semi-open) interval $[t,(t+1)), t \in \mathbb{N}_0^+$. The Information Units (IUs) to be sent over the (single-hop) wireless channel are delivered by the Variable Bit Rate (VBR) Media Encoder of Fig.1 *at the end* of each slot, and they are buffered into a transmit queue of *finite* capacity N_1. Then, they are transferred to a receive playout buffer of finite capacity N_2 (see Fig.1). The number of IUs arriving at the input of the transmit queue at (the end of) slot t is the *connection bandwidth* $\lambda(t) \in \mathbb{R}_0^+$ $(IU/slot)$ of the overall streaming system.

The fading phenomena affecting the wireless link are assumed constant over each slot (i.e., we assume a "block fading" channel) and they are considered independent and identically distributed (i.i.d.) from slot to slot. Thus, the channel-state $\boldsymbol{\sigma}(t) \in \mathbb{R}_0^+$ over slot t is modelled as a real non-negative r.v.[a] with pdf $p_\sigma(\sigma)$. Furthermore, the channel-state value $\sigma(t)$ is assumed to be known at the transmit node at the beginning of slot t, so that, slot by slot, Link-State-Information (LSI) is available at the transmitter of Fig.1. The overall system of Fig.1 is assumed to work in the *steady-state* (i.e., it works under stationary and ergodic operating conditions).

2.1. *Some remarks about the considered media-streaming architecture.*

Before proceeding, some remarks about the media-streaming architecture of Fig.1 may be valuable. Specifically, the above introduced block-fading assumption holds when the coherence-time of the wireless channel of Fig.1 exceeds the time-duration of a slot. Since typical slot-times for

[a]The meaning of the channel-state $\sigma(t)$ is application depending. Without loss of generality and for sake of concreteness, we may consider $\sigma(t)$ be the Signal-to-Disturbance (e.g., the Signal to Noise-plus-Interference) ratio measured at the input of the receive node at the beginning of slot t.

packet-transmission are limited up to 1-2 ms in current 3G/4G wireless delivery systems, then the block-fading assumption may be considered met at medium/low terminal speeds [1], [4], [2].

The i.i.d. assumption on the channel-state random sequence $\{\boldsymbol{\sigma}(t)\}$ is well met in Time-Division-Multiple-Access (TDMA), frequency-hopping or packet-based interleaved delivery-systems of practical interest, where each transmitted IU is independently detected at the receive node [1].

Finally, due to channel reciprocity, the above assumption of LSI at the transmit node of Fig.1 may be considered reasonable in Time-Division-Duplex (TDD) systems, and it is also met in 802.11-based 4GWLANs, where the RTS/CTS handshaking frames can be suitably employed to probe the channel-state [1], [14], [15].

2.2. *Queue's model.*

Let $s(t) \in \mathbb{R}_0^+$ be the number of IUs buffered at the transmit queue of Fig.1 *at the beginning* of slot t. Thus, after denoting by $r(t)$ $(IU/slot)$ the number of IUs to be sent over the wireless channel during the t-th slot (with $r(t) \leq s(t)$), the following Lindley's equation [8]

$$s(t+1) = [s(t) + \lambda(t) - r(t)]^+ , \quad t \geq 0 \tag{1}$$

dictates the evolution of the discrete-time queue-length process $\{s(t) \in \mathbb{R}_0^+, t \geq 0\}$ at the transmit node of Fig.1.

The objective we pursue is the maximization of the (average) transmit rate, by also taking into account for the need to provide to the end user of Fig.1 a playout stream the least bursty as possible (e.g., a Constant Bit Rate (CBR)-like stream) [1]. To this end, after denoting by $q(t) \in \mathbb{R}_0^+$ the number of IUs present in the playout buffer of Fig.1 *at the beginning* of slot t and by $\theta(t)$ $(IU/slot)$ the corresponding number of IUs delivered to the end user over slot t, the playout buffer's evolution equation is given by

$$q(t+1) = [q(t) + r(t) - \theta(t)]^+ , \quad t \geq 0 \tag{2}$$

Without loss of generality, in the following we assume that the IUs sent by the transmit node of Fig.1 over slot t arrive at the playout buffer during slot t and may be scheduled *in the same slot* (that is, we assume that the following constraint holds: $\theta(t) \leq q(t) + r(t)$).

Now, the cost of sending $r(t)$ IUs over slot t is the amount of energy $\mathcal{E}(t)$ (J) required for their transmission. Thus, we may assume that the corresponding number $r(t)$ of IUs sent over the channel during slot t depends on *both* $\mathcal{E}(t)$ and channel-state $\sigma(t)$ via the rate-function $\mathcal{R}(\cdot;\cdot)$ adopted to

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measure the goodput-performance of the considered system, so that we can write

$$r(t) \triangleq \mathcal{R}\left(\mathcal{E}(t); \sigma(t)\right), \quad t \geq 1. \tag{3}$$

From an analytical point of view, $\mathcal{R}(\cdot;\cdot)$ in (3) is a real nonnegative function depending on two nonnegative real arguments $\mathcal{E}(\cdot)$, $\sigma(\cdot)$, and it is measured in $(IU/slot)$. Roughly speaking, $\mathcal{R}(\cdot;\cdot)$ describes the goodput-performance of the DL/PHY layers of the considered system in Fig.1, so that its behavior and analytical properties may depend on several system parameters, such as the requested Quality-of-Service (QoS), the Forward-Error-Correction (FEC) mechanisms implemented at the PHY layer, the fading statistics, the performance of the Automatic-Repeat-reQuest (ARQ)/fragmentation mechanisms possibly implemented at the DL layer, and so on [1]. Therefore, in the sequel we limit to introduce few (quite mild) assumptions on $\mathcal{R}(\cdot;\cdot)$, that, by fact, are retained by the rate-functions of practical interest [7].

First, the rate-function $\mathcal{R}(\mathcal{E};\sigma)$ is continuous on $\mathbb{R}_0^+ \times \mathbb{R}_0^+$ and admits up to second-order continuous derivatives on $\mathbb{R}^+ \times \mathbb{R}_0^+$. Second, it vanishes when either $\mathcal{E} = 0$ or $\sigma = 0$. Third, it is nondecreasing both for $\mathcal{E} \geq 0$ and $\sigma \geq 0$. Fourth, for any assigned $\sigma \neq 0$, the rate-function is assumed to be strictly concave in the $\mathcal{E}$-variable, that is, $\mathcal{R}_{\varepsilon\varepsilon}(\mathcal{E};\sigma) \triangleq \partial^2\mathcal{R}(\mathcal{E};\sigma)/\partial\mathcal{E}^2 < 0$, for $\mathcal{E} > 0$ and $\sigma \neq 0$. Finally, we assume that its first-order derivative $\mathcal{R}_{\varepsilon}(\mathcal{E};\sigma) \triangleq \partial\mathcal{R}(\mathcal{E};\sigma)/\partial\mathcal{E}$ done with respect to the $\mathcal{E}$-argument is nondecreasing in σ for $\sigma \geq 0$.

2.3. *Setup of the tackled optimization problem.*

Let us indicate by $\underline{x}(t) \triangleq [\sigma(t), s(t), q(t)] \in \left(\mathbb{R}_0^+\right)^3$ the overall state of the queue system of Fig.1. Thus, the optimization problem we tackle focuses on the optimal adaptive joint design of the number $r(t)$ of IUs to be sent over the channel at the beginning of slot t (i.e., the scheduling of the *transmit rate*), the number $\lambda(t)$ of IUs to be output by the media encoder of Fig.1 at the end of slot t (i.e., the scheduling of the *connection bandwidth*) *and* the number $\theta(t)$ of IUs to be delivered to the user at the end of slot t (e.g, the scheduling of the playout rate) when $\underline{x}(t)$ is the current system-state.

The ultimate target is the *maximization* of the average transmit rate under physical (PHY), data-link (DL) and application (APP) layer constraints. To formally state the resulting constrained optimization problem, let

$$\mathcal{E}(t) \equiv \varepsilon(\sigma(t); r(t)) \triangleq \mathcal{R}^{-1}(\sigma(t); r(t)), \quad t \geq 1 \tag{4}$$

be the energy requested to send $r(t)$ IUs when the channel-state (e.g., the fading level) is $\sigma(t)$. Therefore, the tackled optimization problem may be formally defined as in the following:

$$\max_{[r(\cdot),\lambda(\cdot)]} \mathrm{E}_{\sigma}\{r(\boldsymbol{\sigma}; s(t); q(t))\} \tag{5}$$

$$s.t.:\ \mathrm{E}_{\sigma}\{\varepsilon(\boldsymbol{\sigma}; r(\boldsymbol{\sigma}; s(t); q(t)))\} \leq \mathcal{E}_{ave} \tag{6}$$

$$0 \leq r(\sigma(t); s(t); q(t)) \leq \min\{s(t); \mathcal{R}(\sigma; \mathcal{E}_p)\} \tag{7}$$

$$0 \leq \lambda(\sigma(t); s(t); q(t)) \leq \lambda_{max} \tag{8}$$

$$0 \leq s(t+1) \leq N_1 \tag{9}$$

$$0 \leq q(t+1) \leq N_2 \tag{10}$$

where (5)-(6) are to be understood as averages (e.g., expectations) conditioned on $\sigma(t)$, while (7)-(10) are instantaneous constraints. Specifically, constraints in (6) and (7) are dictated by the available average energy per slot $\mathcal{E}_{ave}$ (J) and available peak energy $\mathcal{E}_p$ (J) respectively, while (8)-(10) account for the allowed maximum connection bandwidth λ_{max} $(IU/slot)$ and buffer-capacities N_1, N_2 available at the transmit and playout buffers, respectively.

To achieve a proper control of the playout jitter, it is necessary to introduce a memory structure in the scheduler design, so that the controller may exploit the past history of the system for performing traffic-smoothing. To the best of our knowledge, the way the memory has to be incorporated in an optimal joint CL playout controller is still an open problem. Therefore, in the following, we propose a CL approach that allows to optimized overall media-streaming performance at low computational cost. For this purpose, we consider the following playout control strategy $\theta(t)$:

$$\begin{aligned} \theta(t) \equiv \theta(\sigma(t); s(t); q(t)) \equiv \min\Bigg\{ & \frac{1}{1+p} \sum_{i=t-p}^{t} r(\sigma(i); s(i); q(i)); \\ & q(t) + r(\sigma(t); s(t); q(t)) \Bigg\} \end{aligned} \tag{11}$$

that directly depends on the sample average of the last p values of $\{r(t), \ldots, r(t-p)\}$ of the transmit rate, with the integer $p \geq 0$ being the memory-order of the playout policy.

As explicitly pointed out in the following Sect.4, this playout policy constitutes, indeed, an effective (although simple-to-implement) way for reducing the delay-jitter possibly affecting the stream delivered to the end user of Fig.1. To proceed, we introduce eq.(11) into eqs.(5)-(10), so to rewrite the buffer constraints in (9), (10) as in

$$s(t) + \lambda(t) - r(t) \leq N_1$$

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and

$$q(t)+r(t)-\min\left\{\frac{1}{p+1}\sum_{i=t-p}^{t} r(\sigma(i),s(i),q(i));q(t)+r(\sigma(t),s(t),q(t))\right\}\leq N_2$$

respectively. So doing, we obtain for the rates $\lambda(\cdot;\cdot;\cdot)$ and $r(\cdot;\cdot;\cdot)$ to be optimized the following constraints:

$$\lambda(\sigma(t);s(t);q(t))\leq(N_1-s(t))+r(\sigma(t);s(t);q(t)) \tag{12}$$

and

$$r(\sigma(t);s(t);q(t))\leq\left\{\left(\frac{p+1}{p}\right)(N_2-q(t))+\frac{1}{p}\sum_{i=t-p}^{t-1} r(\sigma(i);s(i);q(i))\right\} \tag{13}$$

This allows us, in turn, to recast the constraint in (7) in the following form:

$$0\leq r(\sigma(t);s(t);q(t))\leq r_p(\sigma(t);s(t);q(t)) \tag{14}$$

where

$$r_p(\sigma(t);s(t);q(t))\triangleq\min\left\{s(t);\mathcal{R}(\sigma;\mathcal{E}_p);\left(\frac{p+1}{p}\right)(N_2-q(t))+M_p(t)\right\} \tag{15}$$

is the resulting constraint on the peak throughput, while

$$M_p(t)\triangleq\frac{1}{p}\sum_{i=t-p}^{t-1} r(\sigma(i);s(i);q(i))$$

represents the sliding-window sample average of the transmit rate. In the following section, we derive the expression for the optimal scheduler, while in the Sect.4 we show that this last is able to attain a good tradeoff among the contrasting goals of reducing the (average) delay-jitter present in the delivered stream and maximizing the average rate sent over the wireless channel of Fig.1.

3. The joint controller.

Let us indicate by $\varepsilon_r(r;\sigma)\triangleq\partial\varepsilon(r;\sigma)/\partial r$ the first-order derivative of the energy-function in (4) carried out with respect to the r-argument. Thus, after recognizing that the optimization problem in (5)-(10) is an instance of *convex* optimization problem, the resulting optimal transmit rate $r^{opt}(\cdot;\cdot;\cdot)$, connection bandwidth $\lambda^{opt}(.;.;.)$ and playout rate $\theta^{opt}(.;.;.)$ may be evaluated in *closed-form*, as detailed in the following main *Proposition* 3.1.

Proposition 3.1. *Under the above reported assumptions, the optimal joint controller retains the following basic properties:*

***i*)** *the transmit rate is scheduled according to the following relationship:*

$$\begin{aligned} &r^{opt}(\sigma(t); s(t); q(t)) = \\ &\left[\min\left\{\varepsilon_r^{-1}\left(\sigma; \frac{1}{\mu(s(t); q(t); M_p(t))}\right); \; r_p(\sigma(t); s(t); q(t))\right\}\right]^+ \end{aligned} \tag{16}$$

where $\varepsilon_r^{-1}(\cdot;\cdot)$ denotes the inverse function of $\varepsilon_r(\cdot;\cdot)$ with respect to the $\mathcal{E}$-variable. Furthermore, $\mu(s(t); q(t); M_p(t))$ in (16) is the optimal value of the dual variable of the tackled optimization problem, and it may be computed by solving the following (functional) equation:

$$\int_\sigma \varepsilon(\sigma; r(\cdot))\, p_\sigma(\sigma)\, d\sigma = \mathcal{E}_{ave} \tag{17}$$

where

$$r(\cdot) \triangleq \left[\varepsilon_r^{-1}\left(\sigma; \frac{1}{\mu(s(t); q(t); M_p(t))}\right)\right]_0^{r_p(\sigma(t);s(t);q(t))}$$

***ii*)** *the bandwidth-management is dictated by the following relationship:*

$$\lambda^{opt}(\sigma(t); s(t); q(t)) = \min\left\{\left(N_1 - s(t)\right) + r^{opt}(\sigma(t); s(t); q(t)); \lambda_{max}\right\} \tag{18}$$

***iii*)** *the playout rate is given by*

$$\begin{aligned} &\theta^{opt}(\sigma(t); s(t); q(t)) = r^{opt}(\sigma(t); s(t); q(t)) + \\ &\min\left\{\frac{p}{p+1}\left(M_p(t) - r^{opt}(\sigma(t); s(t); q(t))\right); q(t)\right\}. \end{aligned} \tag{19}$$

Proof. Let $r(\cdot;\cdot;\cdot)$ be any admissible rate scheduler for the problem in eqs.(5)-(10) and let $r^{opt}(\cdot;\cdot;\cdot)$ be the optimal one. In Sect.2, we saw that, after introducing eq.(11) in to eq.(2), the constraint eq.(10) can be rewritten as in eq.(13). Thus, the constraints in eqs.(7) and (13) can be recast in the final form of eq.(14). This means that any admissible scheduler $r(\cdot;\cdot;\cdot)$ must meet the (strictly) convex constraints in eq.(6) and eq.(14) on the available average and peak energies, respectively. In fact, the energy-function (4) can be obtained by inverting the rate-function in (3) with respect to the $\mathcal{E}$-variable, so that, as a direct consequence of the properties retained by $\mathcal{R}(\cdot;\cdot)$, it follows that the energy function is nondecreasing and convex in the r-variable. Furthermore, the problem in eqs.(5)-(10) meets the Slater's conditions for the constraints qualification, so that the optimal scheduler may be obtained via an application of the standard Karush-Khun-Tucker (KKT) conditions [16]. Hence, since the resulting Lagrangian function takes on the following form:

$$\mathcal{L}(r(\sigma;s(t);q(t)),\mu(s(t);q(t);M_p(t))) \triangleq$$
$$\triangleq \int r(\sigma;s(t);q(t))p_\sigma(\sigma)d\sigma - \mu(s(t);q(t);M_p(t))\times$$
$$\times\left(\int \varepsilon(\sigma;r(\sigma;s(t);q(t)))p_\sigma(\sigma)d\sigma - \ \mathcal{E}_{ave}\right),$$

thus, by imposing the condition of vanishing gradient [16], we obtain the relationship: $\varepsilon_r(\sigma(t);r(\sigma(t);s(t);q(t))) = \frac{1}{\mu(s(t);q(t);M_p(t))}$, that allows us, in turn, to arrive at the following expression:

$$r^*(\sigma(t);s(t);q(t)) = \varepsilon_r^{-1}\left(\sigma;\frac{1}{\mu(s(t);q(t);M_p(t))}\right)$$

for the solution of the corresponding unconstrained optimization problem. Therefore, since the Lagrangian function is concave [16], it may be recognized that the solution of the following constrained problem:

$$\max_{0\leq r(\sigma(t);s(t);q(t))\leq r_p(\cdot)} \mathcal{L}(r(\sigma(t);s(t);q(t)),\mu(s(t);q(t);M_p(t))),$$

is the *projection* of $r^*(\sigma(t);s(t);q(t))$ onto the underlying definition set, that is,

$$r^{opt}(\sigma(t);s(t),q(t)) \equiv \left[\varepsilon_r^{-1}\left(\sigma(t);\frac{1}{\mu(s(t);q(t);M_p(t))}\right)\right]_0^{r_p(.)}.$$

This completes the proof of eq.(16). Afterwards, after imposing the complementary condition of the constraint (6), we arrive at

$$\mu(s(t);q(t);M_p(t))\left[\mathrm{E}_\sigma\{\varepsilon(.;.)\} - \mathcal{E}_{ave}\right] = 0,$$

that leads, in turn, to eq.(17) for the computation of $\mu(s(t);q(t);M_p(t))$. Finally, after lumping the constraints in eq.(8) and eq.(12), we directly arrive at the expression in eq.(18) for the optimal controller $\lambda^{opt}(\cdot;\cdot;\cdot)$ of the source rate. □

Before proceeding, we note that for $p = 0$, the optimal schedulers $r^{opt}(\cdot;\cdot)$ and $\lambda^{opt}(\cdot;\cdot)$ just depend on the channel and transmit buffer states $(\sigma(t), s(t))$, while the resulting playout rate $\theta^{opt}(\cdot;\cdot)$ coincides with $r^{opt}(\cdot;\cdot)$. Thus, in this case, the dependence on p disappears and, by fact, we *not have* any playout control. Therefore, in the sequel, $p = 0$ denotes the playout free streaming system. .

4. Performance test and comparison.

In this section, we specialize the above presented (general) results and provide several numerical performance results in terms of average transmit rate, delay and delay-jitter. In the carried out numerical tests, we model the channel-state $\boldsymbol{\sigma}(.)$ as a central χ-squared r.v. with 4-degrees of freedom.

Furthermore, the rate-function $\mathcal{R}(\sigma;\mathcal{E})$ adopted to numerically evaluate the scheduler performance is the following logarithmic one:

$$\mathcal{R}(\sigma;\mathcal{E}) \equiv \log(1+\sigma\mathcal{E}), \quad (IU/slot) \tag{20}$$

commonly employed to measure the so-called Shannon's capacity of transmission channels impaired by Gaussian-distributed additive noise-plus-interference disturbance [1]. Since the energy-function corresponding to (20) is given by

$$\varepsilon(r;\sigma) = \frac{1}{\sigma}(e^r - 1) \tag{21}$$

thus, the (general) expressions in eqs.(15)-(16) specialize to

$$r_p(\sigma(t);s(t);q(t)) = \min\left\{ s(t); \log(1+\sigma\mathcal{E}); \left(\frac{p+1}{p}\right)(N_2+q(t)) + M_p(t) \right\} \tag{22}$$

and

$$r^{opt}(\sigma(t);s(t);q(t)) = \left[\min\left\{ \log\left(\frac{\sigma(t)}{\mu(s(t);q(t);M_p(t))}\right); r_p(\sigma(t);s(t);q(t)) \right\} \right]^+ \tag{23}$$

so that the functional equation in (17) becomes

$$\int_\sigma \left(\exp\left(\left[\log\left(\frac{\sigma(t)}{\mu(s(t);q(t);M_p(t))}\right) \right]_0^{r_p(\sigma(t);s(t);q(t))} - 1 \right) p_\sigma(\sigma) d\sigma = \mathcal{E}_{ave} \tag{24}$$

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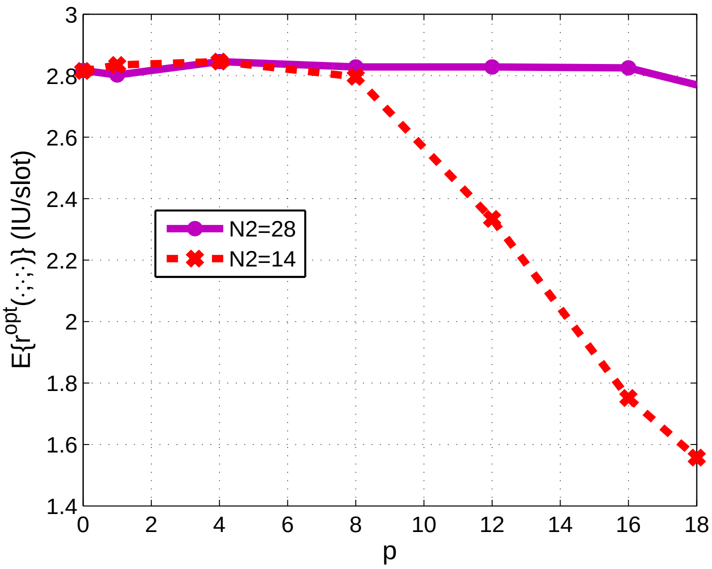

Figure 2. Average transmit rate-vs-p behavior of the optimal controller. $\mathcal{E}_{ave} = 10$ (J), $\lambda_{max} = 3$ $(IU/slot)$, $N_1 = 40$ (IU) are considered.

Hence, we begin to focus on the effect of the memory order p of the playout policy in (11) on the performance of the streaming system of Fig.1. For this purpose, we analyze both the attained average transmit rate and delay. Figs.2-3 refer to the case of $\mathcal{E}_{ave} = 10$ (J), $\lambda_{max} = 3$ $(IU/slot)$, $N_1 = 40$ (IU) and report the performance trends obtained for two values of the playout buffer capacity, namely, $N_2 = 28$ (IU) and $N_2 = 14$ (IU). An examination of the plots of Fig.2 allows us to conclude that, when the playout buffer capacity N_2 is *suitably designed*, the resulting average transmit rate E$\{r^{opt}(\cdot;\cdot;\cdot)\} \triangleq \overline{r}^{opt}$ virtually independent from p (see the continuous line of Fig.2). However, when the capacity N_2 of the playout buffer is not properly designed (see the dotted line in Fig.2), the average transmit rate is not longer independent from the memory order p of the utilized playout policy and the overall performance falls short for increasing value of p. In this case, the introduction of the playout buffer reduces the delay-jitter, but, unfortunately, also reduces the corresponding average transmit rate $\overline{r}^{opt}$.

About the behavior of the plots Fig.2, a final remark is in order. As it could be expected, the introduction of the playout buffer and the resulting additional constraint in terms of finite buffer capacity N_2 tend to decrease

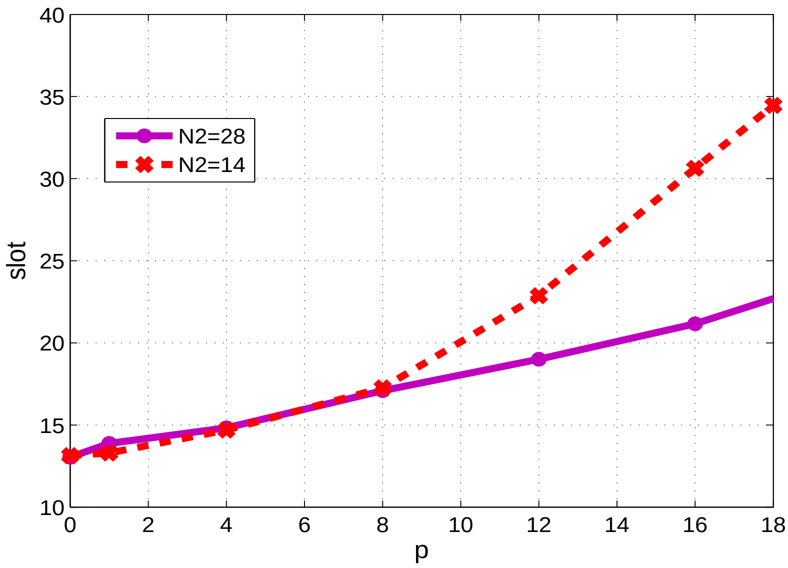

Figure 3. Average delay-vs-p behavior of the joint controller. $\mathcal{E}_{ave} = 10$ (J), $\lambda_{max} = 3$ $(IU/slot)$, $N_1 = 40(IU)$ are considered.

the resulting average transmit rate $\overline{r}^{opt}$ of the overall streaming system. This behavior is due to the fact that the transmit rate $r^{opt}(t)$ must take into account for these additional constraints, so that its degrees of freedom are reduced with respect case of $p = 0$ (e.g., the case of no playout buffer). From Fig.2 we see that, the higher the required memory order p is, the higher the playout capacity N_2 must be, so to avoid substantial reduction in the average transmit rate.

Fig.3 focuses on the corresponding average delays (in multiple of the slot time). An examination of these plots shows that the average delay increases with the memory order p adopted for the playout policy of eq.(11).

Fig.4 reports the corresponding delay-jitters of the transmit rate

$$\sigma_r \triangleq \sqrt{\mathrm{E}\{(\mathrm{r}^{\mathrm{opt}}(\cdot;\cdot;\cdot) - \overline{\mathrm{r}}^{\mathrm{opt}})^2\}}$$

source rate

$$\sigma_\lambda \triangleq \sqrt{\mathrm{E}\{(\lambda^{\mathrm{opt}}(\cdot;\cdot;\cdot) - \overline{\lambda}^{\mathrm{opt}})^2\}}$$

and playout rate

$$\sigma_\theta \triangleq \sqrt{\mathrm{E}\{(\theta^{\mathrm{opt}}(\cdot;\cdot;\cdot) - \overline{\theta}^{\mathrm{opt}})^2\}}$$

when the capacity N_2 of playout buffer is "properly" designed (e.g., $N_2 = 28$ (IU)). Specifically, the plots of Fig.4 show that, even when the im-

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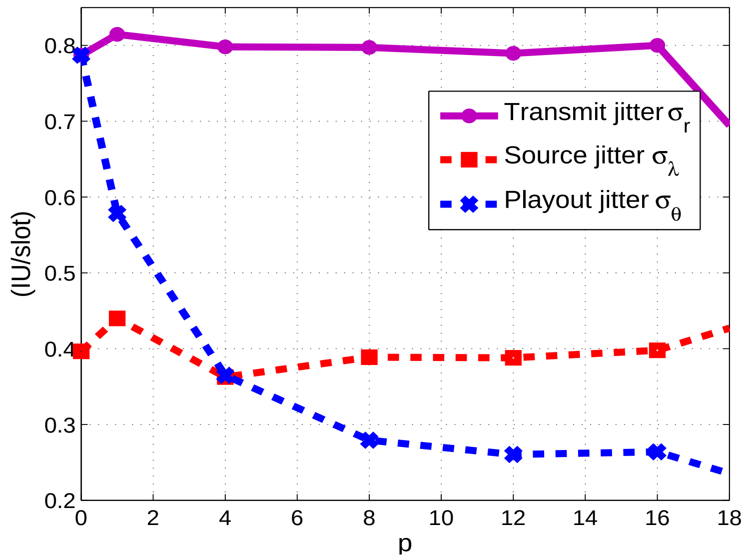

Figure 4. Average jitter-vs-p behavior of the joint controller. $\mathcal{E}_{ave} = 10$ (J), $\lambda_{max} = 3$ $(IU/slot)$, $N_1 = 40$ (IU), $N_2 = 28$ (IU) are considered.

plemented schedulers exhibit the same average transmit rates, the corresponding jitter performance are quite different. Specifically, the transmit rate $r^{opt}(\cdot;\cdot;\cdot)$ presents the higher jitter, due to its tendency to follow the channel-state fluctuations. The rationale behind the introduction of the playout buffer lies, indeed, in the slope of its jitter behavior (see the middle plot of Fig.4): in fact, Fig.4 shows that σ_θ decreases for increasing values of the memory order p of the playout policy in (11). To sum up, the playout policy of eq.(11) is able to reduce the average playout jitter with no effect on the average transmit rate by increasing the resulting average delay values.

Delving further into the impact of the playout buffer capacity N_2 on the performance of the system of Fig.1, let be fixed the memory order of the playout policy in (11) at $p = 12$ and let us analyze the resulting jitter performance via the ratio σ_θ/σ_r. Once a time, Fig.5 supports the conclusion that, beyond a (suitable) threshold value of N_2, the ratio σ_θ/σ_r quickly falls short and then collapses. Hence, this threshold value dictates the boundary among a well-designed system from a bad-designed one. By fact, we could interpret this threshold effect as the price we must pay for providing a CBR-like service to the end user. Nevertheless, from eqs.(16), (15), (17) we deduce that, for increasing values of playout-buffer capacity N_2, the transmit rate becomes less sensitive on the playout dynamics. Furthermore,

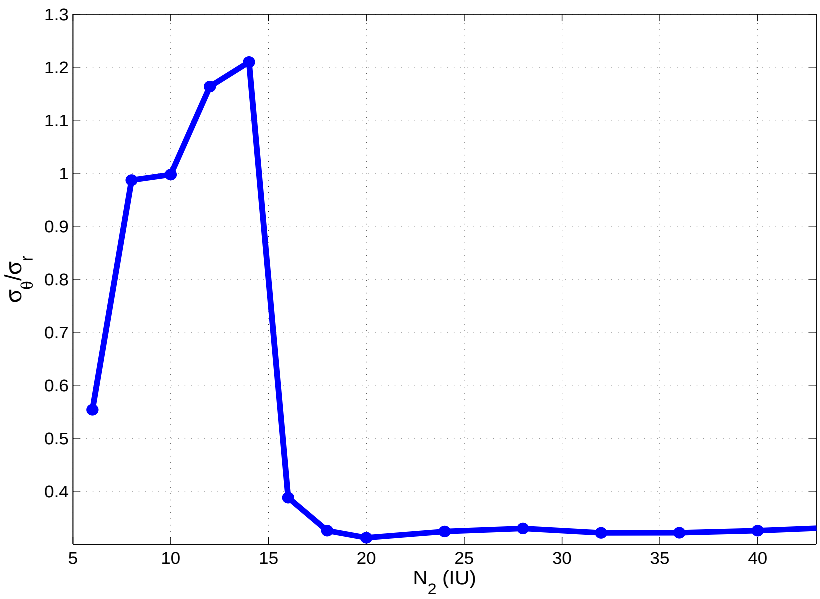

Figure 5. Jitter ratio-vs-N_2 behavior of the optimal controller. $p = 12$, $\mathcal{E}_{ave} = 10$ (J), $\lambda_{max} = 3$ $(IU/slot)$ and $N_1 = 40$ (IU) are considered.

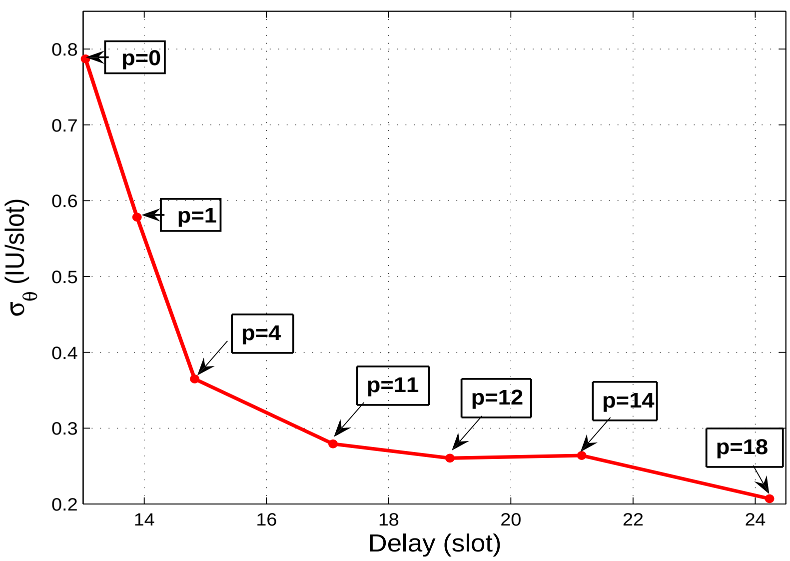

Figure 6. Optimized average playout jitter-vs-delay at $\mathcal{E}_{ave} = 10$ (J), $\lambda_{max} = 3$ $(IU/slot)$, $N_1 = 40$ (IU) and $N_2 = 28$ (IU).

for given channel statistic and system-energy parameters, a minimum value N_2^{min} there exists such that the dual variable $\mu(\cdot;\cdot;\cdot)$ becomes virtually independent from the memory order p of the playout policy. This allows us to conclude that, via a proper tuning of the capacity N_2 of the playout buffer, we are able to strongly mitigate the resulting loss in the average transmit rate. According to this conclusion, the plot of the final Fig.6 may be utilized as an optimized design criterion for linking the attainable delay and delay-jitter performance to the memory order p adopted for the implemented playout policy in (11).

5. Conclusion.

In this paper, we developed an optimized CL solution for the joint adaptive control of the source rate, transmit rate and playout rate for delay and delay-jitter sensitive media-streaming over energy-limited wireless connections. The attained optimized tradeoff among the contrasting goals of high average transmit-rate, low delay and low delay-jitter has been numerically characterized. Furthermore, some related guidelines for an optimized design of the playout policy of eq.(11) and the corresponding playout buffer'capacity have been also derived.

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